





Score	Evaluator	<b>Section III: Work out math questions.</b> (6 questions; 8 points for each; 48 points in total.)

1. The fifth question text text text 0.2 text text, text text 100 text text.

(1) text text text text text text  $\xi$  text 10 text 30 text.

(2) text text text text text text  $\xi$  text 10 text 30 text.

**Solution.**  $E\xi = np = 100 \cdot 0.2 = 20, D\xi = npq = 100 \cdot 0.2 \cdot 0.8 = 16.$  .....2 points

(1)  $P(10 < \xi < 30) = P(|\xi - E\xi| < 10) \geq 1 - \frac{D\xi}{10^2} = 1 - \frac{16}{100} = 0.84.$  .....4 points

(2)  $P(10 < \xi < 30) \approx \Phi_0\left(\frac{30-20}{\sqrt{16}}\right) - \Phi_0\left(\frac{10-20}{\sqrt{16}}\right)$  .....6 points

$= 2\Phi_0(2.5) - 1 = 2 \cdot 0.9938 - 1 = 0.9876$  .....8 points

2. The fourth question, text text  $f = x_1^2 + 2x_1x_2 - 6x_1x_3 + 2x_2^2 - 12x_2x_3 + 9x_3^2$  text text  $f = d_1y_1^2 + d_2y_2^2 + d_3y_3^2$ .

**Solution.**  $f = x_1^2 + 2x_1x_2 - 6x_1x_3 + 2x_2^2 - 12x_2x_3 + 9x_3^2$   
 $= x_1^2 + 2x_1(x_2 - 3x_3) + (x_2 - 3x_3)^2 + x_2^2 - 6x_2x_3$   
 $= (x_1 + x_2 - 3x_3)^2 + x_2^2 - 6x_2x_3$  .....3 points

$= (x_1 + x_2 - 3x_3)^2 + x_2^2 - 2x_2 \cdot 3x_3 + (3x_3)^2 - 9x_3^2$   
 $= (x_1 + x_2 - 3x_3)^2 + (x_2 - 3x_3)^2 - 9x_3^2$  .....6 points

Text  $y_1 = x_1 + x_2 - 3x_3, y_2 = x_2 - 3x_3, y_3 = x_3,$   
 text  $f = y_1^2 + y_2^2 - 9y_3^2$  text. ....8 points

3. The second question  $A(1, 2, -1), B(2, 3, 0), C(3, 3, 2)$  text  $\triangle ABC$  text text text text text.

**Solution.** Text  $\vec{AB} = (1, 1, 1), \vec{AC} = (2, 1, 3)$ , .....2 points

text  $\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = (2, -1, -1)$ , .....4 points

text  $\triangle ABC$  text  $S_{\triangle ABC} = \frac{1}{2} |\vec{AB} \times \vec{AC}| = \frac{1}{2} \sqrt{6}$ . .....6 points

Text text  $2(x-2) - (y-3) - z = 0$ , text  $2x - y - z - 1 = 0$ . .....8 points

4. The third question  $A = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix}$  text.

**Solution.**  $A = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 0 & -1 & -6 & 1 \\ 0 & -6 & -8 & 2 \end{vmatrix} = 1 \cdot (-1)^{2+1} \begin{vmatrix} 1 & 2 & 3 \\ -1 & -6 & 1 \\ -6 & -8 & 2 \end{vmatrix}$  .....4 points

$= - \begin{vmatrix} 1 & 2 & 3 \\ 0 & -4 & 4 \\ 0 & 4 & 20 \end{vmatrix} = - \begin{vmatrix} -4 & 4 \\ 4 & 20 \end{vmatrix} = -(-4 \cdot 20 - 4 \cdot 4) = 96$  .....8 points

5. The sixth question  $N(\mu, \sigma^2)$  text text 16 text, text text text 3160, text text 100. Text text  $H_0 : \mu = 3140$  text text ( $\alpha = 0.01$ ).

- Solution.** (1) Text text  $H_0 : \mu = 3140$ . .....2 points  
 (2) Text text text  $T = \frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t(n - 1)$ . .....3 points  
 (3) Text text  $t_\alpha = t_\alpha(n - 1) = t_{0.01}(15) = 2.947$ . .....5 points  
 (4) Text text text  $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{3160 - 3140}{100/4} = 0.8$ . .....7 points  
 (5) Text  $|t| < t_\alpha$ , text text  $H_0$ , text text text. .....8 points

6. The first question  $\int e^{2x} (\tan x + 1)^2 dx$ .

- Solution.**  $I = \int e^{2x} \sec^2 x dx + 2 \int e^{2x} \tan x dx$  .....2 points  
 $= \int e^{2x} d(\tan x) + 2 \int e^{2x} \tan x dx$  .....4 points  
 $= e^{2x} \tan x - 2 \int e^{2x} \tan x dx + 2 \int e^{2x} \tan x dx$  .....6 points  
 $= e^{2x} \tan x + C$  .....8 points

Score	Evaluator	<b>Section IV: Work out math proofs.</b> (2 questions; 16 points in total.)

1. (7 points) The second question  $A$  text  $B$  text, text  $A$  text  $\bar{B}$  text.

**Proof.**  $P(A \cdot \bar{B}) = P(A - B) = P(A - AB)$  .....2 points  
 $= P(A) - P(AB) = P(A) - P(A)P(B)$  .....4 points  
 $= P(A)(1 - P(B)) = P(A)P(\bar{B})$  .....6 points  
 Text text text  $A$  text text text  $\bar{B}$  text text text. ....8 points

2. (9 points) The first question  $\{x_n\}$  text  $x_1 = \sqrt{2}$ ,  $x_{n+1} = \sqrt{2 + x_n}$ . Text text text, text text text.

**Proof.** (1) Text, text  $x_1 < 2$ , text  $x_k < 2$  text

$$x_{k+1} = \sqrt{2 + x_k} < \sqrt{2 + 2} = 2,$$

Text text text text text  $n$  text  $x_n < 2$ , text text text. Text text

$$\frac{x_{n+1}}{x_n} = \sqrt{\frac{2}{x_n^2} + \frac{1}{x_n}} > \sqrt{\frac{2}{2^2} + \frac{1}{2}} = 1,$$

Text text text text text. Text text text text text, Text text text text. ....4 points

(2) Text text text text  $A$ , text text text text text text

$$A = \sqrt{2 + A}.$$

Text text  $A = 2$ , text text  $\{x_n\}$  text text text 2. ....8 points

**Appendix** Some data may be used in the exam

$\Phi_0(0.5) = 0.6915$	$\Phi_0(1) = 0.8413$	$\Phi_0(2) = 0.9773$	$\Phi_0(2.5) = 0.9938$
$t_{0.01}(8) = 3.355$	$t_{0.01}(9) = 3.250$	$t_{0.01}(15) = 2.947$	$t_{0.01}(16) = 2.921$
$\chi^2_{0.005}(8) = 22.0$	$\chi^2_{0.005}(9) = 23.6$	$\chi^2_{0.005}(15) = 32.8$	$\chi^2_{0.005}(16) = 34.3$
$\chi^2_{0.995}(8) = 1.34$	$\chi^2_{0.995}(9) = 1.73$	$\chi^2_{0.995}(15) = 4.60$	$\chi^2_{0.995}(16) = 5.14$